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Online Appendix

Does a Leading Indicator Related to a Customer Improve a Firm's Profit?

Jumpei Hamamura

Faculty of Business Administration, Momoyama Gakuin University

Online Appendix of
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Proof of Parametric Assumption

In symmetric cases (*i.e.,* $(PI, PI) = (L, L), (NL, NL)$), regardless of the relationship between α and γ, we can obtain the positive outcomes under $\alpha < 1/4$, which is the condition of $\pi_{it}^{(L,L)} > 0$. Additionally, we obtain $\alpha < \gamma$ from $q_{i1}^{(NL,L)} > 0$. ▱

Proof of Observation 3 and 4.

We consider the decision-making of the owner. We identify the optimal strategies in each case where the combinations of performance indicators are $(PI, PI) = (L, L), (NL, L)$ to consider the equilibrium strategy of the performance indicator. First, by backward induction, the optimal strategy in $(PI, PI) = (L, L)$ is as follows:

$$q_{i1}^{(L,L)} = \frac{1}{2 - 2\alpha - \gamma},$$

$$q_{i2}^{(L,L)} = \frac{2 - \gamma - 2\alpha + r(1 + \gamma)}{(3 - 2\alpha)(2 - 2\alpha - \gamma)},$$

$$I_{i}^{(L,L)} = \frac{1}{2 - 2\alpha - \gamma},$$

$$p_{i1}^{(L,L)} = \frac{1 - 2\alpha}{2 - 2\alpha - \gamma}, \qquad \text{(A.1)}$$

$$p_{i2}^{(L,L)} = \frac{(1 - 2\alpha)\big(2 - \gamma - 2\alpha + r(1 + \gamma)\big)}{(3 - 2\alpha)(2 - 2\alpha - \gamma)},$$

$$\pi_{i1}^{(L,L)} = \frac{1 - 4\alpha}{2(2 - 2\alpha - \gamma)^2},$$

$$\pi_{i2}^{(L,L)} = \frac{(1-2\alpha)\big(2-\gamma-2\alpha+r(1+\gamma)\big)^2}{(3-2\alpha)^2(2-2\alpha-\gamma)^2}.$$

When the Parametric Assumption is satisfied, all outcomes are positive.

Next, the optimal strategies and profit in $(PI, PI) = (NL, L)$ are as follows:

$$q_{i1}^{(NL,L)} = \frac{\gamma-\alpha}{\gamma(2-\alpha-\gamma)},$$

$$q_{j1}^{(NL,L)} = \frac{\gamma+\alpha}{\gamma(2-\alpha-\gamma)},$$

$$q_{i2}^{(NL,L)} = \frac{(1-\alpha)\gamma(2-\alpha-\gamma)+r\big(\gamma(1+\gamma)+\alpha^2(1-\gamma)-\alpha(3-2\gamma+\gamma^2)\big)}{(3-\alpha)\gamma(2-\alpha-\gamma)},$$

$$q_{j2}^{(NL,L)} = \frac{(1+\alpha)\gamma(2-\alpha-\gamma)+r\big(\gamma(1+\gamma)-\alpha^2(1-\gamma)+\alpha(3-2\gamma+\gamma^2)\big)}{(3-\alpha)\gamma(2-\alpha-\gamma)},$$

$$I_{i}^{(NL,L)} = \frac{\gamma-\alpha}{\gamma(2-\alpha-\gamma)},$$

$$I_{j}^{(NL,L)} = \frac{\gamma+\alpha}{\gamma(2-\alpha-\gamma)}, \qquad \text{(A.2)}$$

$$p_{i1}^{(NL,L)} = \frac{\gamma-\alpha}{\gamma(2-\alpha-\gamma)},$$

$$p_{j1}^{(NL,L)} = \frac{\gamma(1-2\alpha)+\alpha}{\gamma(2-\alpha-\gamma)},$$

$$p_{i2}^{(NL,L)} = \frac{(1-\alpha)\gamma(2-\alpha-\gamma)+r\big(\gamma(1+\gamma)+\alpha^2(1-\gamma)-\alpha(3-2\gamma+\gamma^2)\big)}{(3-\alpha)\gamma(2-\alpha-\gamma)},$$

$$p_{j2}^{(NL,L)} = \frac{(1-\alpha)\gamma(2-\alpha-\gamma)+r(\gamma(1+\gamma)-\alpha^2(1-\gamma)+\alpha(3-4\gamma-\gamma^2))}{(3-\alpha)\gamma(2-\alpha-\gamma)},$$

$$\pi_{i1}^{(NL,L)} = \frac{(\gamma-\alpha)^2}{2\gamma^2(2-\alpha-\gamma)^2},$$

$$\pi_{j1}^{(NL,L)} = \frac{(\gamma+\alpha)(\gamma(1-4\alpha)+\alpha)}{2\gamma^2(2-\alpha-\gamma)^2},$$

$$\pi_{i2}^{(NL,L)} = p_{i2}^{(NL,L)}q_{i2}^{(NL,L)},$$

$$\pi_{j2}^{(NL,L)} = p_{j2}^{(NL,L)}q_{j2}^{(NL,L)}.$$

When the Parametric Assumption is satisfied, all outcomes are positive.

From Eqs. (11), (A.1), and (A2), we obtain $\Pi_i^{(NL,L)} - \Pi_i^{(L,L)}$ and $\Pi_i^{(NL,NL)} - \Pi_i^{(L,NL)}$ as follows:

$$\Pi_i^{(NL,L)} - \Pi_i^{(L,L)} = -\frac{2\alpha D}{(3-2\alpha)^2(3-\alpha)^2\gamma^2(2-\alpha-\gamma)^2(2-2\alpha-\gamma)^2},$$
$$\Pi_i^{(NL,NL)} - \Pi_i^{(L,NL)} = -\frac{2\alpha E}{9(3-\alpha)^2(2-\gamma)^2\gamma^2(2-\alpha-\gamma)^2}, \quad \text{(A.3)}$$

where $D \equiv 3(2-\gamma)\gamma\big(27-46\gamma+15\gamma^2+6\gamma^3-\gamma^4+r^2(18-4\gamma-9\gamma^2+12\gamma^3-\gamma^4)+2r(18-29\gamma+24\gamma^2-9\gamma^3+\gamma^4)\big)-4\alpha^7(1+2r^2(1-\gamma)^2+4r(1-\gamma)\gamma+2\gamma^2)+4\alpha^6\big(11+\gamma+20\gamma^2-6\gamma^3+4r\gamma(11-12\gamma+3\gamma^2)+r^2(22-42\gamma+26\gamma^2-6\gamma^3)\big)-\alpha^5\big(193+48\gamma+312\gamma^2-204\gamma^3+26\gamma^4+4r\gamma(201-242\gamma+108\gamma^2-13\gamma^3)+r^2(386-708\gamma+540\gamma^2-212\gamma^3+26\gamma^4)\big)-\big(432+233\gamma+547\gamma^2-668\gamma^3+185\gamma^4-12\gamma^5+2r\gamma(992-1321\gamma+798\gamma^2-189\gamma^3+12\gamma^4)+r^2(864-1518\gamma+1421\gamma^2-752\gamma^3+185\gamma^4-12\gamma^5)\big)-\alpha^3\big(522+585\gamma+233\gamma^2-994\gamma^3+492\gamma^4-70\gamma^5+2\gamma^6+2r\gamma(1432-2111\gamma+1546\gamma^2-536\gamma^3+$

$70\gamma^4 - 2\gamma^5) + 2r^2(522 - 847\gamma + 991\gamma^2 - 671\gamma^3 + 254\gamma^4 - 35\gamma^5 + \gamma^6)\big) - 3\alpha\big(27 + 189\gamma - 293\gamma^2 + 70\gamma^3 + 78\gamma^4 - 38\gamma^5 + 4\gamma^6 + 2r\gamma(186 - 343\gamma + 317\gamma^2 - 162\gamma^3 + 41\gamma^4 - 4\gamma^5) + 2r^2(27 + 3\gamma + 44\gamma^2 - 82\gamma^3 + 69\gamma^4 - 22\gamma^5 + 2\gamma^6)\big) + \alpha^2(324 + 801\gamma - 600\gamma^2 - 516\gamma^3 + 570\gamma^4 - 144\gamma^5 + 9\gamma^6 + 2r\gamma(1212 - 1987\gamma + 1662\gamma^2 - 735\gamma^3 + 146\gamma^4 - 9\gamma^5) + r^2(648 - 822\gamma + 1343\gamma^2 - 1224\gamma^3 + 666\gamma^4 - 148\gamma^5 + 9\gamma^6))$, and $E \equiv 3(2-\gamma)\gamma(27 - 46\gamma + 15\gamma^2 + 6\gamma^3 - \gamma^4 + r^2(18 - 4\gamma - 9\gamma^2 + 12\gamma^3 - \gamma^4) + 2r(18 - 29\gamma + 24\gamma^2 - 9\gamma^3 + \gamma^4)) + \alpha^3(9 - 45\gamma + 16\gamma^2 + 11\gamma^3 - 5\gamma^4 - r\gamma^2(2 + \gamma - \gamma^2) + 2r^2(9 - 27\gamma + 29\gamma^2 - 14\gamma^3 + 2\gamma^4)) - \alpha^2(54 - 288\gamma + 169\gamma^2 + 42\gamma^3 - 54\gamma^4 + 10\gamma^5 - r\gamma(36 - 16\gamma + 51\gamma^2 - 48\gamma^3 + 11\gamma^4) + r^2(108 - 324\gamma + 346\gamma^2 - 171\gamma^3 + 24\gamma^4 + \gamma^5)) + \alpha(81 - 513\gamma + 466\gamma^2 - 65\gamma^3 - 102\gamma^4 + 46\gamma^5 - 5\gamma^6 - r\gamma(180 - 250\gamma + 266\gamma^2 - 195\gamma^3 + 71\gamma^4 + -- 10\gamma^5) + r^2(162 - 522\gamma + 526\gamma^2 - 263\gamma^3 + 15\gamma^4 + 25\gamma^5 - 5\gamma^6))$. Based on the numerical example ($r = 0.1$ and $\gamma = 0.2$), we consider $\Pi_i^{(NL,L)} - \Pi_i^{(L,L)}$ and $\Pi_i^{(NL,NL)} - \Pi_i^{(L,NL)}$. In this case, we obtain $D \simeq 22.932 - 178.4489\alpha + 497.241\alpha^2 - 691.5768\alpha^3 + 531.872\alpha^4 - 228.678\alpha^5 + 51.182\alpha^6 - 4.627\alpha^7 > 0$ and $E \simeq 22.932 - 5.6469\alpha - 3.2804\alpha^2 + 0.804\alpha^3 > 0$. From this analysis, we obtain $\Pi_i^{(NL,L)} < \Pi_i^{(L,L)}$ and $\Pi_i^{(NL,NL)} < \Pi_i^{(L,NL)}$. This implies that both firms adopt the leading indicator under specific conditions.

In addition, we consider $\Pi_i^{(NL,NL)} - \Pi_i^{(L,L)}$ as follows:

$$\Pi_i^{(NL,NL)} - \Pi_i^{(L,L)} = \frac{2\alpha F}{9(3-2\alpha)^2(2-\gamma)^2(2-2\alpha-\gamma)^2}, \tag{A.4}$$

where $F \equiv 3(70 - 113\gamma + 51\gamma^2 - 8\gamma^3 + \gamma^4) + 4\alpha^3(17 - 8\gamma + 2\gamma^2) - \alpha^2(52 + 60\gamma - 8\gamma^3) - \alpha(199 - 404\gamma + 132\gamma^2 + 4\gamma^3 - 2\gamma^4) - 2r(2 + \gamma - \gamma^2)(8\alpha^3 + 8\alpha^2(5-\gamma) + 2\alpha(19 - 7\gamma + \gamma^2) - 3(2 + \gamma - \gamma^2)) + r^2(1+\gamma)^2(8\alpha^3 - 8\alpha^2(5 -$

$\gamma) + 2\alpha(37 - 16\gamma + \gamma^2) - 3(8 - 2\gamma - \gamma^2))$. When we specify r and γ as $r = 0.1$ and $\gamma = 0.2$, we obtain $F \simeq 145.0068 - 107.2932\alpha - 81.07776\alpha^2 + 65.49\alpha^3 > 0$ under $0 < \alpha < 1/4$. From this outcome, there exists the case in which $\Pi_i^{(NL,L)} < \Pi_i^{(L,L)}$, $\Pi_i^{(NL,NL)} < \Pi_i^{(L,NL)}$, and $\Pi_i^{(NL,NL)} > \Pi_i^{(L,L)}$ hold under the specific condition. $\square$